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A novel form of the MHD Rayleigh–Taylor instability

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When a mutually perpendicular horizontal magnetic field and electric current are imposed upon an electrolytic liquid in a tank covered by a lighter non-conducting fluid, the interface can become unstable. In the experiment described in this paper the tank floor is stepped so that only part of the interface becomes unstable. The result is a distinctive arch-like configuration of the electrolyte, which finally disintegrates to break the current path, but which will continue to repeat itself.

1. Introduction

When a conducting liquid with a free surface is subject to an electromagnetic body force produced by imposing a horizontal magnetic field **B** perpendicular to a horizontal electric current **j**, waves excited on the surface propagate anisotropically (Shercliff 1969; Robinson 1973). If the body force acts upwards and is sufficient to overcome gravity, the surface becomes unstable, a situation known as the magnetohydrodynamic (MHD) Rayleigh–Taylor instability. The significant feature of the MHD Rayleigh–Taylor instability is that it is also an anisotropic phenomenon, there being a preferential direction for the wavenumber vector describing the mode of instability.

Lemaire (1963) and Duc (1968) have shown theoretically that the preferred mode of instability is for the surface to deform into corrugations of growing amplitude having troughs and crests lying parallel to the direction of the imposed current and normal to the imposed magnetic field; see figure 1. This can be attributed to the surface boundary condition on the current, which dictates that the $\mathbf{j} \times \mathbf{B}$ force must be normal to the surface for wavelike disturbances with crests parallel to the y direction and vertical for crests parallel to the x direction. In the former case the $\mathbf{j} \times \mathbf{B}$ force cannot contribute to the stability conditions, whereas in the latter case the $\mathbf{j} \times \mathbf{B}$ force, if acting upwards, competes with gravity and tends to destabilize the surface.

This theoretical result has been supported by the experiments of Baker (1965) and Duc (1968). First interest in the problem derived from consideration of striated MHD generators, where bands of unseeded, less conducting gas are alternated in a duct with bands of seeded, better conducting gas, and the instability can occur at the interfaces between layers. The problem has more general interest, however, in a subject like MHD, where there are still many areas to be more fully investigated. This is one of those situations where an

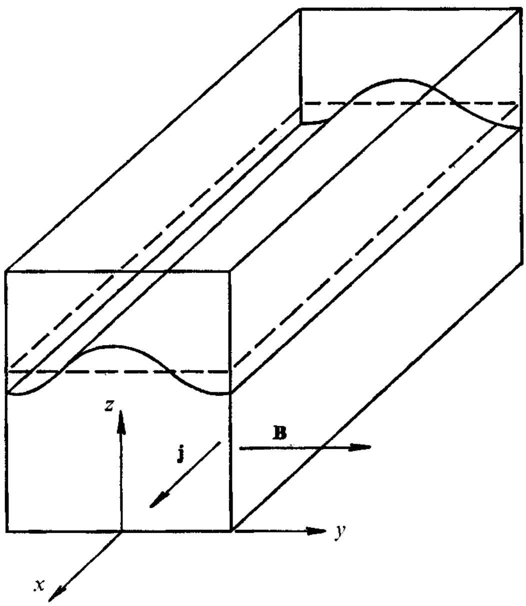

FIGURE 1. Orientation of instability.

ordinary hydrodynamical phenomenon is modified by the direct imposition of $\mathbf{j} \times \mathbf{B}$ forces, the resulting motion falling into the fairly small class of MHD problems which can be studied both theoretically and with relative ease in the laboratory. The results described here arose from a theoretical and experimental investigation into the effects on the MHD Rayleigh–Taylor instability of varying the configuration of the imposed magnetic field and electric current, described more fully in Robinson (1973). In particular, this novel form of instability was observed when the current density was caused to vary in the direction of current flow by means of variation in the depth of the containing vessel.

2. Theoretical considerations

To describe the actual experimental conditions, we shall consider the stability of an interface between two fluids, one being electrically conducting with density ρ_2 and the other an insulator of density ρ_1. ρ_2 is taken to be greater than ρ_1, so that with no electromagnetic force present the insulator is stably at rest above the conducting fluid.

The stability equation for the MHD Rayleigh–Taylor instability (Lemaire 1963) is then

$$-n^2 = \frac{(\rho_2-\rho_1)\,gk + k^3\alpha - kB_y j_x}{\rho_1+\rho_2},$$

where $\zeta = Z_0 e^{nt+iky}$ is the representation of the interface displacement shown in figure 1. α is the interfacial tension. Clearly the interface is unstable when

$$B_y j_x > (\rho_2-\rho_1)\,g + k_{\min}^2\,\alpha,$$

$k_{\min}$ being the minimum wavenumber possible, corresponding to one wavelength between the walls of the containing vessel.

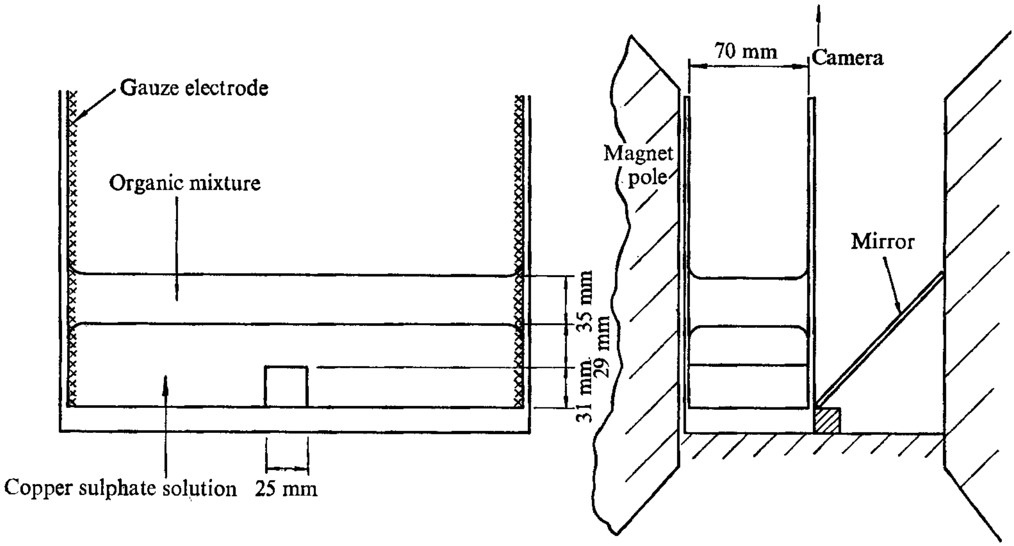

FIGURE 2. Arrangement of experimental tank.

If the electromagnetic parameters are to be varied, it is necessary to impose constraints so that the fluids still remain in equilibrium under the new configuration of forces. This requires that there be no rotational forces, which cannot be balanced by pressure gradients within the fluid, i.e. $\operatorname{curl} \mathbf{j} \times \mathbf{B} = 0$. Since

$$\operatorname{div} \mathbf{j} = \operatorname{div} \mathbf{B} = 0$$

this condition becomes

$$(\mathbf{B}.\nabla)\mathbf{j} - (\mathbf{j}.\nabla)\mathbf{B} = 0.$$

If we let $\mathbf{B}$ remain uniform and in the y direction, then the constraint becomes

$$(\mathbf{B}.\nabla)\mathbf{j} = 0, \quad \text{or} \quad \partial\mathbf{j}/\partial y = 0.$$

Thus it is possible to vary the current in the x, z plane, provided that such variation is the same at all y within the fluid. Clearly, the containing vessel must be rectangular with side walls parallel to the x and y axes. The simplest situation to envisage where the current distribution is non-uniform is that shown in figure 2, where the tank floor is stepped. The current density in the shallower part of the conducting fluid must be greater than that in the deeper part.

Now let us consider the case where $\mathbf{j}$ is sufficiently great in the shallow part for $j_x B_y > (\rho_2 - \rho_1) g + k_{\min}^2 \alpha$ there, whereas this is not so in the deeper part. There will then be a tendency towards instability in the shallower part, but not the deeper. This raises questions as to the stabilizing effect of the deeper region on the shallower, the motion which occurs at the boundary between stable and unstable regions, and the possibility of a quasi-stable state in which the surface is deformed, fluid motion is induced, but dynamic equilibrium is achieved between the rotational forces now operating (i.e. the electromagnetic $(\mathbf{B}.\nabla)\mathbf{j}$ forces balance the viscous forces of the induced motion). Such a situation is difficult to analyse theoretically, involving three-dimensional viscous MHD flow without the simplifications of high Hartmann number analysis. Hence a simple experiment was mounted to enable practical observations to be made. Some evidence was obtained for the existence of such a quasi-stable state, and is described in Robinson (1973), but it is the form of the large amplitude development of the instability which proves to be the most interesting and spectacular phenomenon and which is described here.

Mention must also be made of the influence of surface or interfacial tension on Rayleigh–Taylor instability experiments. Assuming that the interface is stable, its two-dimensional equilibrium shape in the Oyz plane is governed by

$$\frac{(\rho_2-\rho_1)g-j_xB_y}{\alpha}z_0-\frac{\partial_2 z_0/\partial y^2}{[1+(\partial z_0/\partial y)^2]^{\frac{3}{2}}}=0,$$

where z_0 is the height of the interface above its level at a position remote from the wall and $\mathbf{j}\times\mathbf{B}$ is assumed to be acting upwards. For the purpose of this discussion of the meniscus action we are considering a tank with no step in it, i.e. j_x is uniform. In the experiment described later the same argument applies in principle but a complete analysis of the meniscus action would have to take into account the variation of j_x in the z direction. The boundary conditions are $\partial z_0/\partial y = \tan\theta$ at the wall, where θ is the angle of contact, and $\partial z_0/\partial y \rightarrow 0$ as $y\rightarrow\infty$. This is the familiar meniscus effect (see, for example, Batchelor 1967, p. 66) with the electromagnetic term included. If h is the height of the meniscus above (or below) the interface level at $y\rightarrow\infty$, then

$$h^2=\frac{2\alpha(1-\sin\theta)}{(\rho_2-\rho_1)g-j_xB_y}.$$

Now when the MHD Rayleigh–Taylor instability is being produced experimentally, it is generally necessary that the magnitude of j_xB_y is increased, either gradually or suddenly. Consequently h will increase and the interface will require a new shape if it is to be in static equilibrium with the increased $\mathbf{j}\times\mathbf{B}$ force. This is a purely hydrostatic effect and must not be mistaken for the onset of the instability itself. If j_xB_y is slowly increased, $h\rightarrow\infty$ as $j_xB_y\rightarrow(\rho_2-\rho_1)g$ and in practice the meniscus will reach to the bottom or top of the containing vessel. If j_xB_y is suddenly increased to a destabilizing magnitude, then part of the initial motion which is observed will be due to this movement of the meniscus towards an equilibrium position, not to the actual instability mechanism. Consequently it should be taken into account in any experimental measurement of initial growth rates. This effect should not be confused with the role of surface tension in tending to make a surface stable to disturbances of shorter wavelengths. It is purely a meniscus effect and will in fact not be present if the angle of contact is 90°.

3. Observation of the instability

The experiment was performed in an electromagnet having water-cooled copper coils and an iron core with pole faces of area 12 × 6 in., which could produce a field of up to 0·7 Wb/m² (using the available power) uniform to within

5 % across the gap between the pole faces. A rectangular Perspex tank was built, with a stepped-up floor in its central region as shown in figure 2. Electric current was supplied from a d.c. power supply which could operate as a constant-current or constant-voltage source. Copper gauzes were used for the electrodes, since these tended to give a more uniform current density than copper plates, and the conducting fluid chosen was copper sulphate solution. To enable the $\mathbf{j} \times \mathbf{B}$ force to compete with the gravity force without resorting to large current densities, with consequent overheating and electrode corrosion problems, the copper sulphate was covered by a non-conducting mixture of 1, 1, 1 trichloroethane and olive oil in proportions making the density of the mixture just less than that of the electrolyte. White spirit was used instead of olive oil for other experiments, but for this particular case the phenomenon occurred more slowly and was therefore easier to photograph sequentially when the more viscous olive oil was used. The depths of each fluid were as shown in figure 2. The tank was mounted at one side of the magnet gap so that a mirror could be placed as shown to enable both the side and top view of the motion to be observed simultaneously from above. The progress of the motion was photographed using a Shackman auto-camera, set to take a photograph every second. With the upper fluid at a specific gravity of 1·152, the electrolyte at a specific gravity of 1·162 and the magnetic field at 0·4 Wb/m² a current of 0·6 A was switched on between the electrodes. This produced a value of 118 N/m³ for $j_x B_y$ in the shallow region and 57 N/m³ in the deep region, compared with $(\rho_2 - \rho_1) g + \alpha k^2_{\min} \approx 102$ N/m³, α being taken as 0·04 N/m³ from measurements of the meniscus rise in a capillary tube with opposite ends open to the upper and lower fluid respectively. The state of the interface at given times after switch-on is recorded in the photographs in figure 3 (plates 1 and 2).

Initially the surface slowly changed shape, presumably under the influence of the surface tension as described above, causing the meniscus to increase in size and to reach down towards the floor of the tank in the shallow region, as can be seen clearly in the plan views in figures 3(*b*) and (*c*), although it does not show up against the dark electrolyte in the side views. This continued whilst the true instability gathered momentum, so that the position where the interface touched the wall of the tank reached the bottom corner as the interface began to bulge upwards in the middle. The displacement of the interface produced a flow within the electrolyte (observable at the time through the movement of small particles of impurities) which tended to stabilize the motion and if the current was set at a lower magnitude the state observed at $t = 4$ s could apparently be maintained without further development of the instability.

On allowing the instability to develop further, the intersection of the interface and the tank was seen to move along the floor of the tank until it linked with the interface on the other side, forming a circle of the lighter fluid right round the copper sulphate, which then formed a 'bridge' between two bodies of stable electrolyte on either side of the stepped floor. In the side view in figure 3(*d*) the lighter fluid can just be seen to pass under the conducting fluid bridge. This bridge then carried all the current passing through the tank, and the $\mathbf{j} \times \mathbf{B}$ force lifted the bridge bodily through the lighter fluid. Because of the nature of the MHD Rayleigh–Taylor instability the fluid interface in this bridge was still stable to 'pinch' type modes of instability, and remained stable in shape. However, as it was lifted, two other effects occurred. The ends were anchored to the main bodies of fluid, so that the bridge arched upwards in the middle, and this permitted fluid to run out down either end as if it were a hose, since whilst the cylindrical shape as a whole was stable, there were no forces acting to prevent the fluid flowing under gravity down each side. The other effect was that distortion of current in the two main bodies of fluid produced rotational $\mathbf{j} \times \mathbf{B}$ forces which induced motion such that the ends of the bridge were carried away from the middle and each other, thus stretching the bridge. As the bridge narrowed in cross-section, its resistance increased and the current generator could no longer operate in a constant-current mode, but switched to constant voltage, consequently reducing the current. The bridge finally became so narrow that it broke, presumably under surface-tension forces, and once the current path was broken the whole bridge broke up into bubbles which floated slowly back to the bottom. After some time the two bodies of fluid met again and coalesced, and if the current source was still on, the instability would repeat itself indefinitely in a graceful motion.

4. Outstanding questions

At first sight this motion might be compared with the original pinch experiments of Northrup (1907), where a metallic conductor in a trough was made to pinch itself into two parts by the current passing through it, with no imposed field, and hence a similar intermittent current flow was obtained. However, in that case, the pinch was due to the radial pressure gradient produced by interaction of the current and its own induced field, whereas in our case, using electrolyte, the current densities are not high enough for this effect to be at all significant, even just before the breaking of the bridge, so that the pinch mode in which the bridge appears to break must be due to surface-tension forces.

This paper attempts merely to demonstrate the existence of an interesting instability, but the observations as they stand raise many questions which must be answered if a full understanding of the phenomenon is to be gained. The relative ease with which the instability can be produced experimentally suggests that further parallel theoretical and experimental investigations should be possible. There appear to be four distinct parts to the motion which could be studied separately in more detail.

(*a*) *Steady dynamic equilibrium.* The stable state observed at sufficiently low currents, which could be maintained without proceeding to the instability, was the subject of the original investigation which led to the discovery of the bridge instability. It has been suggested that a solution for this dynamic equilibrium should be possible by separation of variables, if the equations are linearized. This would be valid only for low speeds, but should reveal the essential characteristics of the three-dimensional flow field. The physical basis of a solution would be a balance between rotational viscous and $\mathbf{j} \times \mathbf{B}$ forces, the latter being the result of the deformed interface, which itself would be determined by the pressure distribution within the conducting fluid. It would be advisable to seek a solution first for the case where the conducting fluid is not covered by a lighter one, in which there would be no tangential stresses acting at the surface. The geometric form of the tank floor will affect both the motion itself and probably the ease of solution. It could be that a ramp or sinusoidal profile would be more tractable than a step. The aim of such analysis should be to determine which parameters govern the dynamic equilibrium motion and the conditions under which dynamic equilibrium could be maintained without the onset of instability. It should be possible to test the results of analysis by experimental observation of the disturbed surface profile. This could be done optically without interfering with the motion. Direct measurement of the velocity field within the fluid would be a much harder experimental undertaking.

(*b*) *Meniscus effects.* The movement of the interface in response to surface-tension effects as the body force is varied can be studied independently of the instability, enabling its contribution to the total unstable motion to be determined. The introduction of MHD forces makes possible a time-variable body force, turning the meniscus effect, which is classically a hydrostatic problem, into a hydrodynamic problem. In the context of the instability it would be important to determine the characteristic time required for the interface to assume a new form, and which physical parameters govern the transient motion. To discover the general characteristics of the meniscus-dominated flow, a two-dimensional solution would be sufficient, considering a simple rectangular tank and uniform j_x. A further interesting problem, however, is to determine the static equilibrium interface shape for the situation where there is a step in the tank and j_x is not uniform. This would have much in common with the three-dimensional meniscus problem studied by Concus & Finn (1969), except that the body force would be varying in space. It would be necessary to assume that any surface distortions which led to non-zero $\partial \mathbf{j}/\partial y$ were small enough for the resulting rotational forces to have a negligible effect.

(*c*) *The formation of the bridge configuration.* The next distinct stage of the instability requiring analysis is the creeping of the meniscus along the tank floor resulting in the encircling of the conducting liquid and the formation of the bridge. For the case of a long step, with a region in its centre where the problem could be considered two-dimensional, an analytical description of the motion should be possible, drawing on the study of the transient meniscus motion suggested above. This should enable a characteristic time to be determined for this stage of the instability, and it would be worthwhile attempting to determine the stage, if any, after which the instability cannot be stabilized and must inevitably pursue its course.

Another important consideration at this stage of the instability is the effect of the geometry of the tank. It was observed in a much wider tank than the one described here that several bridges formed across the width of the tank (Robinson 1973). In that case the characteristic wavelength of the Rayleigh–Taylor instability was much less than the tank width, and presumably it was as the troughs of the instability reached the tank floor that the bridge-forming mechanism was initiated. The implication is that, although the final stage of bridge formation is due to meniscus creep, the typical width of the bridge when first formed must be related to the Rayleigh–Taylor wavelength appropriate to the fluid parameters above the step. Another geometric parameter of importance is the ratio of the depth of the conducting fluid to the step height, since it governs the ratio between the $\mathbf{j} \times \mathbf{B}$ forces in the stable and unstable regions and hence the buoyancy force obtainable in the forming bridge. A third geometric ratio to be considered is that of the depth of the conducting fluid above the step to the step length. This is large when the step is a thin barrier, and it is not obvious that a bridge would form at all in that case. A comprehensive experimental programme is necessary to isolate the individual importance of these various geometrical parameters in the bridge-forming mechanism.

(*d*) *The motion and collapse of the bridge*. The last stage of the instability which can be distinctly analysed is the behaviour of the conducting fluid bridge itself. Once again the geometric parameters of the step length and the depth of the conducting fluid are obviously important factors governing the dynamics of the bridge. The depth of the non-conducting fluid is also important now, since in the above observations it can be seen that the bridge is inhibited from rising further by the upper surface of the fluid. In a non-conducting fluid of much greater depth the bridge might not be so flat in cross-section and as the instability developed it might tend to arch higher, with a consequent change of lifetime. Further experiments are necessary to determine the influence of these geometric parameters.

Analytically the outstanding questions to be answered are as follows.

(i) Is analysis possible of the motion in the main body of conducting fluid at either end of the tank which causes the movement of the ends of the bridge away from each other?

(ii) What is the relative importance of the two effects causing the bridge to narrow, i.e. the stretching of the bridge and the flow of fluid out under gravity? Is it possible to determine the form and motion of the bridge analytically, perhaps for a simplified case such as the central, almost straight, region of a long bridge?

(iii) What governs the life-span of the bridge? Unsuccessful attempts were made by adjusting the current input to maintain the bridge indefinitely. Is this theoretically possible? This would involve consideration of whether the bridge is stable to pinch modes of perturbation.

(iv) What is the mechanism of final breakup of the bridge? Does it depend solely on surface-tension effects once the bridge has become thin enough, or do the electromagnetic forces play a part in helping or hindering the final collapse?

In conclusion it can be observed that the number of interesting questions raised by this instability is illustrative of the character of low magnetic Reynolds number MHD, where the presence of a variable body force can introduce many interesting variations into classical OHD problems.

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REFERENCES

Baker, R. C. 1965 Maximum growth rates of Rayleigh–Taylor instabilities due to an electromagnetic force. *Nature*, **207**, 65–66.

Batchelor, G. K. 1967 *An Introduction to Fluid Dynamics*. Cambridge University Press.

Concus, P. & Finn, R. 1969 On the behaviour of a capillary surface in a wedge. *Proc. Nat. Acad. Sci. U.S.A.* **63**, 292–299.

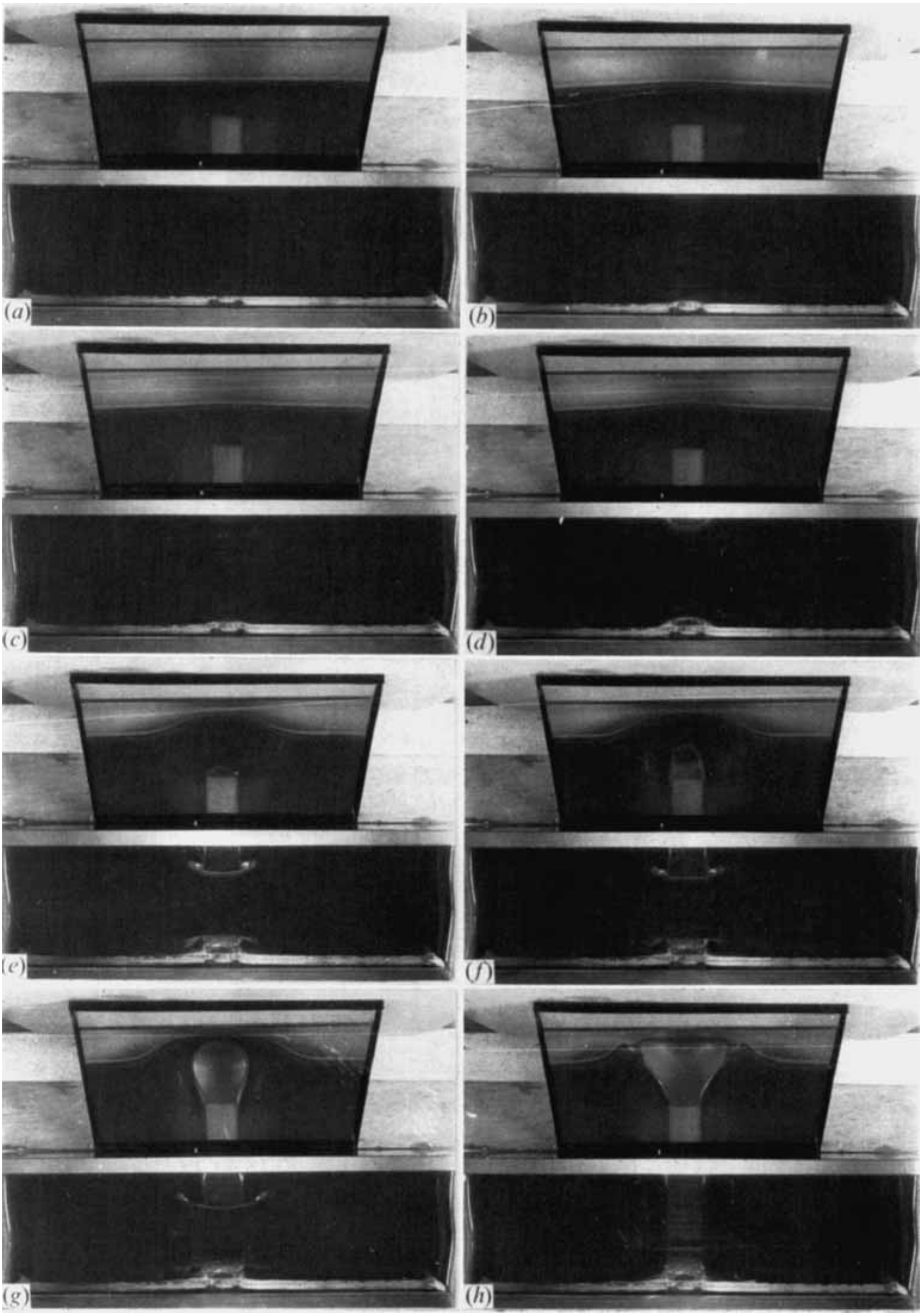
Duc, X. N. 1968 Etude théorique et experimentale sur modèle hydraulique des instabilités de Rayleigh–Taylor en magneto hydrodynamique. *C. r. Acad. Sci. Paris*, **A 266**, 738–740.

Lemaire, A. 1963 Instabilité de Rayleigh–Taylor sous l'effet de forces electromagnetiques. In *Comptes Rendus de la IVe Conference Internationale sur les Phenomènes d'Ionisation dans les Gaz* (ed. P. Hubert & E. Cremaieu-Alcon), vol. 4, pp. 487–490. Paris: SERMA.

Northrup, E. F. 1907 Some newly observed manifestations of forces in the interior of an electric conductor. *Phys. Rev.* **24**, 474–497.

Robinson, I. S. 1973 Anisotropic surface waves and instabilities under a vertical electromagnetic force. Ph.D. thesis, University of Warwick.

Shercliff, J. A. 1969 Anisotropic surface waves under a vertical magnetic force. *J. Fluid Mech.* **38**, 353–364.



FIGURES 3 (*a–h*). For legend see over.

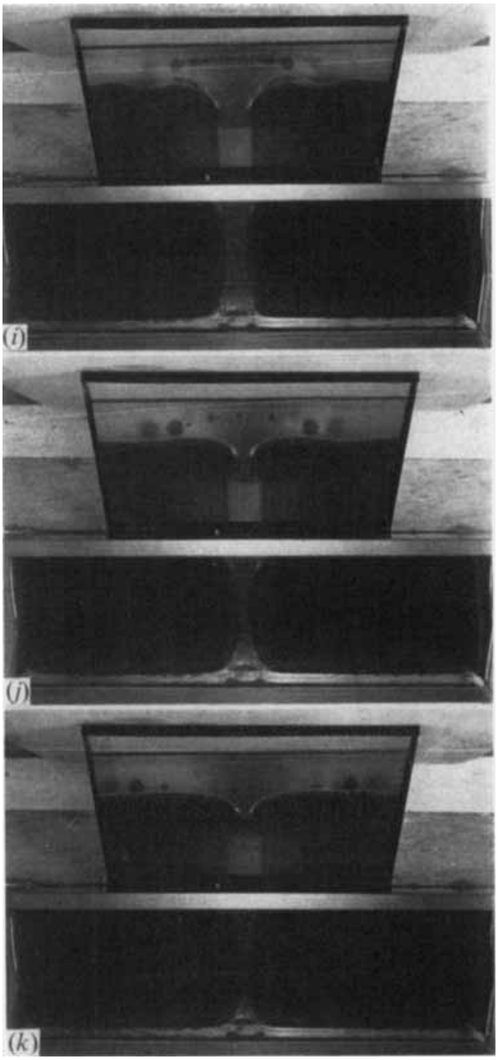


FIGURE 3. Photographs of instability at stated times after current switch-on. (*a*) 0 s. (*b*) 4 s. (*c*) 6 s. (*d*) 9 s. (*e*) 14 s. (*f*) 15 s. (*g*) 16 s. (*h*) 17 s. (*i*) 18 s. (*j*) 19 s. (*k*) 23 s.